REPORT DOCUMENTA AD-A255 974

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
4/1/92

3. REPORT TYPE AND DATES COVERED
Annual Progress Report 10/1/90-9/31/91

4. TITLE AND SUBTITLE
Delamination Growth Behavior in Cross-Ply Laminated Composites Due to Transverse Concentrated Loading (u)

5. FUNDING NUMBERS
AFOSR 89-0554

(2)

6. AUTHOR(S)
Sheng Liu and Fu-Kuo Chang

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Stanford University
Aeronautics & Astronautics
Stanford, CA 94305-4035

8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR 92 0891

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR / NA
Bolling, Washington DC 20331-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

DTIC ELECTE OCT 07 1992 S A D

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)
An analytical model was developed to study the delamination growth behavior of graphite/epoxy cross-ply laminated composites resulting from quasi-static transverse concentrated loads. The objective of the study was to fundamentally understand the interaction between the initial matrix cracking and the delamination initiation and growth in laminated composites due to transverse concentrated loads.

During this period of the investigation, attention is given to the growth of the surface matrix crack-induced delamination in cross-ply composites and the interaction between the matrix cracking and the delamination initiation and propagation inside the laminates. Based on the study, it shows that delamination growth in laminated composites due to transverse loads is strongly affected by the initial matrix cracks. Delamination induced by a bending crack (surface) would grow into a slender shape with its major axis parallel to the direction of the surface cracks. Mode I fracture toughness dominates the initiation and growth of the delamination induced by the bending cracks. Also, bending crack-induced delamination is stable in laminated composites.

14. SUBJECT TERMS
Laminated Composites, Delamination Growth, Transverse Concentrated Loading

15. NUMBER OF PAGES
25

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

DELAMINATION GROWTH BEHAVIOR IN CROSS-PLY LAMINATED COMPOSITES DUE TO TRANSVERSE CONCENTRATED LOADING

Annual Progress Report
for the period of October 1990 to September 1991
AFOSR Contract Number 89-0554

submitted to

Dr. Walter Jones
Director of Mechanics of Materials
Air Force Office of Scientific Research
Bolling, Washington DC 20331-6448

by

Sheng Liu* and Fu-Kuo Chang**
Department of Aeronautics and Astronautics
Stanford University
Stanford, CA 94305

* Graduate Student
** Associate Professor

ABSTRACT

An analytical model was developed to study the delamination growth behavior of graphite/epoxy cross-ply laminated composites resulting from quasi-static transverse concentrated loads. The objective of the study was to fundamentally understand the interaction between the initial matrix cracking and the delamination initiation and growth in laminated composites due to transverse concentrated loads.

The model consists of a stress analysis for determining the stress distributions during loading, a contact analysis for modeling the interfaces conditions of matrix cracks and delaminations, and a failure analysis for predicting crack growth. The stress analysis was developed based on the up-dated Lagrange formulation. A nonlinear three-dimensional finite element analysis was developed for calculating the stresses inside the composites. An augmented Lagrangian method was adopted in the contact analysis for modelling the crack interfaces. No friction was considered along the crack surfaces. The indentation resulting from a spherical nose indenter was also modelled based on the contact method. The crack closure technique based on linear elastic fracture mechanics was applied to determine the delamination propagation.

During this period of the investigation, attention is given to the growth of the surface matrix crack-induced delamination in cross-ply composites and the interaction between the matrix cracking and the delamination initiation and propagation inside the laminates. Based on the study, it shows that delamination growth in laminated composites due to transverse loads is strongly affected by the initial matrix cracks. Delamination induced by a bending crack (surface crack) would grow into a slender shape with its major axis parallel to the direction of the surface cracks. Mode I fracture toughness dominates the initiation and growth of the delamination induced by the bending cracks. The analysis also showed that the growth of the bending crack-induced delamination is stable in laminated composites.

TABLE OF CONTENTS

I. INTRODUCTION

It is well known that fiber-reinforced organic matrix laminated composites are vulnerable to transverse quasi-static or transient dynamic loading. Matrix cracking and delaminations are commonly observed failure modes appeared inside the materials resulting from transverse quasi- or low-velocity impact. Extensive studies have been performed by many investigators and can be found in the literature, however, the majority of the previous work related to the damage induced by transverse loading has been focused primarily on experiments [$1-16$].

It has been shown in the literature that for some ply orientations, matrix cracks could be generated without initiating delaminations by properly selecting the impact velocity or applied load. However, whenever there was a delamination, there were matrix cracks accompanied with the delamination. Recent studies by Jones [15], Joshi [10], and Sun [4] indicated from their experiments that matrix cracking was the initial failure mode for organic matrix laminated composites subjected to transverse loading.

Choi and Chang [$17-19$] recently performed an experimental and analytical study on line-loading impact. The results of the study also showed that the damage in composites resulting from transverse loads was initiated from matrix cracking. Furthermore, these initial cracks were found to play a much more significant and crucial roles in the damage development due to transverse loads than those matrix cracks generated commonly by in-plane tensile loads. Unlike the matrix cracks generated by in-plane loads, these initial matrix cracks, once produced, would induce immediately delaminations along the interfaces where the two neighboring plies have different ply orientations. Basically, these initial matrix cracks can be classified into two types: the shear matrix crack and the bending crack. The shear cracks appear inside the laminates and locate at a distance away from the impacted area. The bending cracks occur on the bottom ply of the laminate directly beneath the impact load.

From their experiments, Choi and Chang [$17-19$] showed that the damage induced by the shear cracks was much more violent and catastrophic than that initiated by the bending cracks. It has been observed that the size and shape of the delaminations inside composites were strongly dependent upon the location and the type of the initial matrix cracks.

Apparently, the strong interaction between the initial matrix cracking and delamination growth is a major characteristic of the damage in composites resulting from transverse loads. Such a strong interaction sets apart the major difference between the damage induced by in-plane loads and transverse loads. Unfortunately, there is limited understanding of the mechanics of the material governing the interaction and the crack propagation. Recently, Sun [20] performed a two-dimensional study on the delamination growth induced by a bending crack. Mode I fracture was found to be the major factor contributing to the delamination growth.

Therefore, the objective of this investigation was to study the interaction between matrix cracking and delamination growth in laminated composites resulting from transverse loads. In the first year of the study, a nonlinear two-dimensional finite element analysis was developed to study delamination propagation induced by both shear matrix cracks and bending cracks. The results ot the study showed that the position and type of the initial matrix cracking significantly affected the growth of delamination. Delamination growth could be either stable or unstable depending upon the location of the initial matrix cracks. The study characterized the basic failure modes and failure mechanism of laminated composites resulting from transversely concentrated loading in two dimensions [21,22].

In the second year of the study, the attention was focused on understanding delamination growth in three dimensions in laminated composites under a single transverse concentrated load. The objective was to develop a model for analyzing matrix crack-induced delamination growth in laminated composites subjected to transversely concentrated loading through a spherical indenter. Liu [23] modelled an embedded delamination with a thin soft isotropic layer employing a finite element method. No matrix cracking was considered in the analysis. It was shown that Mode II and III fractures controlled predominantly the growth of the embedded delamination due to transverse loading.

Due to the complexity of the problem, only cross-ply laminates were considered. A three dimensional finite element analysis was developed during the investigation. During this period, only bending crack-induced delaminations were studied. In the third year work has been continued to evaluate the delaminations induced by shear matrix cracks. In this report, the analytical model will be briefly described. The primary focus will be

given on the discussion of the effect of initial matrix cracking on the propagation of the delamination in composites, and a detailed description of the analysis will be provided in the final report at the end of the third year study.

II. PROBLEM OF STATEMENT

Consider a $[0_6/90_2]_s$ T300/976 laminated composite plate subjected to a quasi-static transverse concentrated loading as shown in Figure 1. A spherical indenter was applied vertically downward to the center of the laminate. It was assumed that a surface matrix crack in the bottom 0° ply group initiated the damage, followed by a delamination in the second 90/0 interface measured from the top surface of the laminate. A typical damage pattern of the laminate from an X-radiograph is shown in Figure 2.

In this investigation, it was desired to determine

1). the effect of the initial matrix cracking on the propagation of the delamination in the plate,

2). the delamination growth in the laminate at a given load, and

3). the response of the delaminated composite.

Accordingly, the laminate containing a delamination with and without a pre-introduced matrix crack in the 0° ply group was analyzed as shown in Figure 3. A three-dimensional finite element method was developed for analyzing the problem. A brief description of the analysis will be given in the next section.

III. MODEL

Because the local indentation deformation of the laminate could be substantial due to concentrated loading, the finite deformation theory [24] was adopted in the analysis. The total potential energy Π of the laminated composite subjected to the external loading can be described in general as

$$\Pi = \int_v W(S_{ij}, E_{ij})\, dv - \int_A \overline{T}_i u_i \, dA \tag{1}$$

where W is the strain energy function, and S_{ij} and E_{ij} are the total Kirchhoff Stresses and the Green Strains, respectively. $\overline{T}_i$ are the prescribed surface tractions applied on the current deformed surface A, and u_i are the displacements.

In order to analyze the response of the delaminated plate subjected to transverse concentrated loading, resulting from a spherical indenter, the contact conditions between the upper and lower delamination surfaces inside the material, and between the indenter and the laminate have to be modelled accurately. Because both conditions can affect significantly the deformed configuration of the laminate, Eq. (1) can not be solved alone without prior knowledge of the conditions. However, these contact conditions are unknown and depend upon the deformed configuration of the composite.

In general, the following normal contact conditions have to be satisfied along the contact surface for both delamination interface and the indenter, as shown in Figure 4:

$$g \geq 0 \tag{2.1}$$

$$\lambda_n \leq 0 \tag{2.2}$$

where λ_n is the normal contact force, and g is a gap function which defines the distance between the upper and lower delamination surfaces or between the indenter and the laminate.

Eq. (2.1) represents the impenetrability condition, and Eq. (2.2) indicates that the normal contact tractions must be compressive between contacting bodies. Accordingly, combining Eqs. (2.1) with (2.2) yields

$$\lambda_n g = 0 \tag{3}$$

Eqs. (2) and (3) represent the well-known Kuhn-Tucker conditions [25]. The above conditions should be held at any configuration of the laminate at a given load.

Therefore, Eq. (1) also has to satisfy the constraints. In general, there are two methods that are widely used for enforcing the constraints: 1) Lagrange multiplier method

[26] and 2) penalty method [27, 28]. The former has the advantage of enforcing the exact constraints, but introduces additional parameters and substantially enlarges the system of equations to be solved [26]. This method has been applied by the authors previously for analyzing the interaction between matrix cracking and delamination in two dimensions [21, 22]. However, it was found very difficult to be applied for the present three dimensional problem which resulted in a very large system of equations.

The latter has the advantage of introducing no additional equations, but it is very sensitive to the choice of penalty parameter, possibly leading to ill-conditioning [27, 28]. Recently, an augmented Lagrangian method [29], a mixed formulation between the Lagrangian multiplier and penalty methods, was developed, which showed to circumvent the ill-conditioning problem and preserve the advantage of the Lagrangian multiplier method without introducing additional variables. Therefore, in this study, the augmented Lagrangian formulation was adopted. Thus, based on the augmented Lagrangian formulation, by combining Eqs. (1) with (3) yields

$$\delta\Pi + \int_{\Gamma}(\lambda_n + \epsilon_n g)\delta g d\Gamma = 0 \tag{4}$$

where λ_n has to be up-dated within each load increment until the gap function is minimized. ϵ_n is the penalty parameter and Γ is the contact surface. It is noted that if $g = 0$, then λ_n in above equation corresponds to the correct Lagrange multiplier.

A three dimensional finite element method was developed based on Eqs. (3) and (4), and an eight-node brick element was used. Due to symmetry, only a quarter of the laminate was analyzed. A typical finite element mesh is shown in Figure 5.

Once the deformations were calculated from the finite element analysis, the strain energy release rates along the delamination front were determined based on the crack closure techniques [30]. Only the delaminations with circular or elliptical shapes were considered in the analysis as shown in Figure 6. In order to predict the delamination growth, a three dimensional crack growth criterion was required. Numerous crack growth criteria have been proposed in the literature. Here, the following crack growth criterion [31] was adopted:

$$\frac{G_I}{G_{IC}} + \frac{G_{II}}{G_{IIC}} + \frac{G_{III}}{G_{IIIC}} = E_d \tag{5}$$

If $E_d \geq 1$, then delamination growth occurs. In this study, $G_{IIC} = G_{IIIC}$ was assumed [31] for the material considered. A similar criterion has been used previously by the authors [21, 22] for analyzing the delamination growth in two dimensions.

IV. RESULTS AND COMPARISONS

In order to verify the model, numerical solutions were generated and compared with existing analytical and numerical solutions. Overall, the agreements were very good. Verification of the analysis will not be presented here but will be provided in the final report. In the following, numerical simulations of a cross-ply laminate containing a delamination with and without a surface matrix crack will be presented and discussed.

Figure 7 shows the response of the laminate containing an elliptical delamination and subjected to transverse loading. The load-deflection response was linear at the initial loading and became stiffened as the load increased. However, the contact area between the indenter and the laminate was highly nonlinear starting from initial loading as shown in the figure.

In order to evaluate the effect of surface matrix cracking on the delamination initiation and propagation, numerical simulations were performed on a T300/976 graphite/epoxy $[0_6/90_2]_s$ composite subjected to a transversely quasi-static loading resulting from a spherical indenter. Numerical calculations were generated for the laminate containing various sizes of a pre-introduced delamination with and without a surface matrix crack in the bottom 0° ply-group. The length of the matrix crack was assumed to be a constant throughout the calculations. In the following figures, numerical results will be presented for delaminations from small to large sizes.

Figure 8 shows the comparisons of the calculated total strain energy release rate G_T for the laminate containing a small circular delamination with and without the surface matrix crack. For the laminate containing the matrix crack, the value of G_T along the delamination front near the location of the matrix crack increased significantly and reached a peak at the intersection between the matrix crack and delamination (at $\phi =$

0°). However, it decreased rapidly as the delamination front, where the value of G_T was calculated, was away from the location of the matrix crack (ϕ approaches to 90°).

A completely different distribution of the total strain energy release rate was obtained for the laminate containing no pre-introduced matrix crack. Overall, the values of G_T were much smaller than those calculated from the laminate containing the matrix crack. Apparently, the laminate with the matrix crack could initiate the delamination growth at a much earlier loading stage than the one without. Once the delamination propagates, it would grow along with the direction of matrix cracking.

Figure 9 shows the sequence of the delamination growth in the laminate with the matrix crack predicted from the model on the basis of the three dimensional crack growth criterion (Eq. (5)). The number shown in the bracket at the upper right corner of each sub-figure indicates the sequence. The sub-figures were generated by evaluating the strain energy release rate ratio E_d for various sizes of delaminations. First, the values of E_d were calculated for a small circular implanted delamination. If delamination growth was predicted ($E_d \geq 1$), then the delamination size was extended slightly along its major (X-direction, $\phi = 0°$) or minor (Y-direction, $\phi = 90°$) axis according to the predicted direction of delamination growth. In this analysis, the delamination was only allowed to grow into either a circular or an elliptical shape. Again numerical calculations were then re-performed to evaluate the strain energy release rates for the laminate with the new delamination. This procedure was repeated until the calculated strain energy release rate ratio was smaller than unity everywhere along the delamination front. The size of the final delamination was then considered to be the delamination shape corresponding to the given loading condition.

At the fixed indenter displacement, Figures 9-1 and 9-2 show that the initial delamination grew from a small circular shape into a slender elliptical shape along the direction at $\phi = 0°$ with its major axis parallel to the fiber direction of the bottom ply group. The delamination growth was unstable because the strain energy release rates actually increased as the delamination expanded. Figure 9-3 indicates the delamination would continue to expand along its major axis, but the strain energy release rate ratio started to decrease along the delamination front near $\phi = 0°$.

After a substantial growth of the delamination along its major axis, Figure 9-4

shows that the strain energy release rates started to increase near $\phi = 75°$ away from the matrix crack, initiating the expansion of the delamination into the minor axis. However, a slight expansion of the delamination along its minor axis as shown in Figure 9-5 would subsequently cause a significant increase of the strain energy release rates along the major axis. Hence, the delamination started to grow along the major axis again. Apparently, the growth of the delamination was predominantly controlled by the delamination front near the neighborhood where the matrix crack intersected with the delamination. It is worth noting that at $\phi = 90^o$, strain energy release rates remained to be minimum regardless of the shape of the delamination. This implies that no delamination would grow in this area, leading the delamination to a peanut shape which was frequently observed from the experiments for this type of ply orientation such as the one given in Figure 2.

However, a completely different growth pattern was predicted for the laminate containing no matrix crack. First, at the same load, no delamination growth was predicted for the laminate containing the same small delamination as given in Figure 9-1. An additional displacement (load) must be applied in order to initiate the delamination growth. At 0.05 inches of indenter displacement, Figure 10.1 shows that delamination growth initiated at $\phi = 0°$. As the delamination expanded in its major axis from Figure 10-1 to 10-2, the strain energy release rate increased, indicating an unstable growth. It is noted that as the delamination continued to expand in its major axis, the peak of the strain energy release rate ratio gradually shifted more and more toward $\phi = 90$ degree, indicating a much more uniform expansion of delamination. Figure 10-3 indicates delamination expansion in the minor axis due to high strain energy release rates near $\phi = 90°$. The distribution of the strain energy release rate ratio along the delamination front for all the delaminations was relatively uniform compared to that of the laminate with the matrix crack. As a result, the delamination tended to grow into an elliptical shape with major and minor axis ratio about 2. However, for the same laminate containing a pre-introduced matrix crack, such ratio was close to 5.

Figure 11 shows the calculated strain energy release rates of Mode I, II, and III for the laminate containing a delamination with and without a matrix crack. Clearly, for the laminate containing the initial matrix crack, G_I (Mode I fracture) dominated the total strain energy release rate along the delamination front near the neighborhood where the

surface matrix crack intersected with the delamination ($\phi = 0°$). However, Modes II and III fractures dominated the total strain energy release rate for the laminate without the matrix crack. Although the contribution of each Mode on the growth of the delamination strongly depends upon the current shape of delamination, the presence of the matrix crack clearly play a very important role on the delamination growth. Therefore, it is very important that the initial matrix crack must be considered in the analysis for understanding the damage mechanics and mechanism of laminated composites due to transverse loading.

IV. CONCLUDING REMARKS

A finite element analysis was developed for analyzing cross-ply laminated composite plates subjected to transverse concentrated loading resulting from a spherical indenter. Based on the analysis, the following remarks can be made for the laminates studied:

1). the initial matrix cracking affects significantly the growth of delamination resulting from transverse loading.

2). the surface matrix crack induces the delamination to grow into a slender peanut shape.

3). Mode I fracture toughness contributes significantly to the growth of the delamination induced by a bending crack.

Additionally, it is noted that the proposed finite element analysis can also be extended to study other ply orientations and other loading conditions, such as delamination-buckling.

VI. ACKNOWLEDGEMENT

The support of the Air Force Office of Scientific Research for the investigation is greatly appreciated. Dr. Walter Jones was the Contract Monitor. The computations were performed at National Science Foundation Computing Center at University of California, San Diego.

VII. REFERENCES

1) Sjöblom, P.O., J. T. Hartness, and T. M. Cordell, "On Low-Velocity Impact Testing of Composite Materials," *J. of Computers and Structures, Vol. 22*, 1988, pp. 30-52.

2) Clark, G., "Modelling of Impact Damage in Composite Laminate," *Composites, Vol. 20*, 1988, pp. 209-214.

3) Poe, Jr., C.C., "Simulated Impact Damage in a Thick Graphite/Epoxy Laminate Using Spherical Indenters," NASA TM-100539, January, 1988.

4) Joshi, S.P. and C. T. Sun, "Impact-Induced Fracture in Quasi-Isotropic Laminate," *J. of Composite Technology and Research, Vol. 19*, 1986, pp. 40-46.

5) Finn, S.R. and G. S. Springer, "Composite Plates Impact Damage: An Atlas," Technomic Publishing Co., Inc., Lancaster, USA, 1991.

6) Liu, D., and L. E. Malvern, "Matrix Cracking in Impacted Glass/Epoxy Plates," *J. of Computers and Structures, Vol. 21,* 1987, pp. 594-609.

7) Finn, S. R. "Delaminations in Composite Plates under Transverse Static or Impact Loads," Ph.D. Thesis, Department of Aeronautics and Astronautics, Stanford, University, 1991.

8) Wu, H. T., and G. S. Springer, "Measurements of Matrix Cracking and Delamination Caused by Impact on Composite Plates," *J. of Composite Materials, Vol. 22,* 1988, pp. 518-532.

9) Gosse, J. H., and P. B. Y. Mori, "Impact Damage Characterization of Graphite/Epoxy Laminates," Proceedings of the Third Technical Conference of the American Society for Composites, Seattle, WA, 1988, pp. 334-353.

10) Joshi, S. P., "Impact-Induced Damage Initiation Analysis: An Experimental Study," Proceedings of the Third Technical Conference of the American Society for Composites, Seattle, WA, 1988, pp. 325-333.

11) Abrate, S., "Impact of Laminated Composite Materials - a Review," *Composites, Vol. 22,* 1991, pp. 347-362.

12) Cantwell, W. J., and J. Morton, "The Impact Resistance of Composite Materials," *Applied Mechanics Review, Vol. 44,* 1991, pp. 155-190.

13) Poe, Jr., C.C., "Simulated Impact Damage in a Thick Graphite/Epoxy Laminate Using Spherical Indenters," NASA TM-100539, January, 1988.

14) Malvern, L. E., R. L. Sierakowski, L. A. Ross, and N. Cristescu, "Impact Failure Mechanisms in Fiber-Reinforced Composite Plates," *High Velocity Deformation of Solids* (Kawata and Shiori, eds.), Springer-Verlag, Berlin, 1978, pp. 120-130.

15) Jones, S. J. Paul, T. E. Tay, and J. F. Williams, "Assessment of the Effect of Impact Damage in Composites: Some Problems and Answers," *Composite Structures Vol. 10,* 1988, pp. 51-73.

16) Guynn, E. G., and T. K. O'Brien, "The Influence of Lay-up and Thickness on Composite Impact Damage and Compression Strength," Proceedings of AIAA/ASME/ASCE/AHS 26th Structure, Structural Dynamics and Materials Conference, Orlando, FL, 1985, pp. 187-196.

17) Choi, H.Y., R. J. Downs, and F. K. Chang, "A New Approach Toward Understanding Damage Mechanism and Mechanics of Laminated Composites Due to Low-Velocity Impact, Part I- Experiments," *J. of Composite Materials, Vol. 25,* 1991, pp. 992-1011.

18) Choi, H. T., H. Y. T. Wu, and F. K. Chang, "A New Approach Toward Understanding Damage Mechnisms and Mechanics of Laminated Composites Due to Low-Velocity Impact, Part II - Analysis," *J. of Composite Materials, Vol. 25,* No. 8, 1991, pp. 1012-1038.

19) Choi, H. Y., H. S. Wang, and F. K. Chang, "Effect of Laminate Configuration and Impactor's Mass on the Initial Impact Damage of Composite Plates Due to Line-Loading Impact," *J. of Composite Materials,* (to appear).

20) Sun, C. T., and M. G. Manoharan, "Growth of Delamination Cracks Due to Bending in a $[90_5/0_5/90_5]$ Laminate," *Composite Science and Technology, Vol. 34,* 1989, pp. 365-377.

21) Liu, S., Z. Kutlu, and F. K. Chang, "Matrix Cracking-Induced Stable and Unstable Delamination Propagation in Graphite/Epoxy Laminated Composites due to Quasi-Static Transverse Concentrated Load," *Composite Materials: fatigue and Fracture (4th Symposium),* 1991.

22) Liu, S., Z. Kutlu, and F. K. Chang, "Matrix Cracking and Delamination Propagation in Laminated Composites Subjected to Transversely Concentrated Loading," *Journal of Composite Materials,* (submitted) 1991.

23) Lu, X. and D. Liu, "Strain Energy Release Rates at Delamination Front," Proceedings of the American Society of Composites, Fourth Technical Conference, Oct. 3-5, 1989, pp. 277-286.

24) Bethe, K.J., "Finite Element Procedures in Engineering Analysis," Prentice-Hall, Inc., New Jersey, 1982.

25) Kikuchi, N. and J. T. Oden, Contact Problems in Elasticity: A Study of Variational Inequalities and Finite element Methods, SIAM, Philadelphia, 1988.

26) Bethe, K.J. and A. Chaudhary, "A Solution Method for Planar and Axisymmetric Contact Problems," *International J. for Numerical Methods in Engineering, 21,* 1985, pp. 65-88.

27) Benson, D.J. and J. O. Hallquist, "A Single Surface Contact Algorithm for the Post-buckling Analysis of Shell Structures," *Computer Methods in Applied Mechanics and Engineering, 78,* 1990, pp. 141-163.

28) Parisch, H., "A Consistent Tangent Stiffness Matrix for Three-dimensional Non-linear Contact Analysis," *International J. for Numerical Methods in Engineering, 28,* 1989, pp. 803-1812.

29) Simo, J.C. and T. A. Laursen, "An Augmented Lagrangian Treatment of Contact Problems Involving Friction," *Computers and Structures Vol. 42,* No.1, 1992, pp. 97-116.

30) Rybicki, E.F.and M. F. Konninen, "A Finite Element Calculation of Stress Intensity Factors by a Modified Crack Closure Integral," *Engineering Fracture Mechanics, Vol.*

9, 1977, pp. 931-938.

31) Chai, H.,"Mixed-Mode Fracture Behavior of Delaminated Films," Proceeding of the American Society for Composites, Fourth Technical Conference, Oct. 3-5, 1989, pp. 815-825.

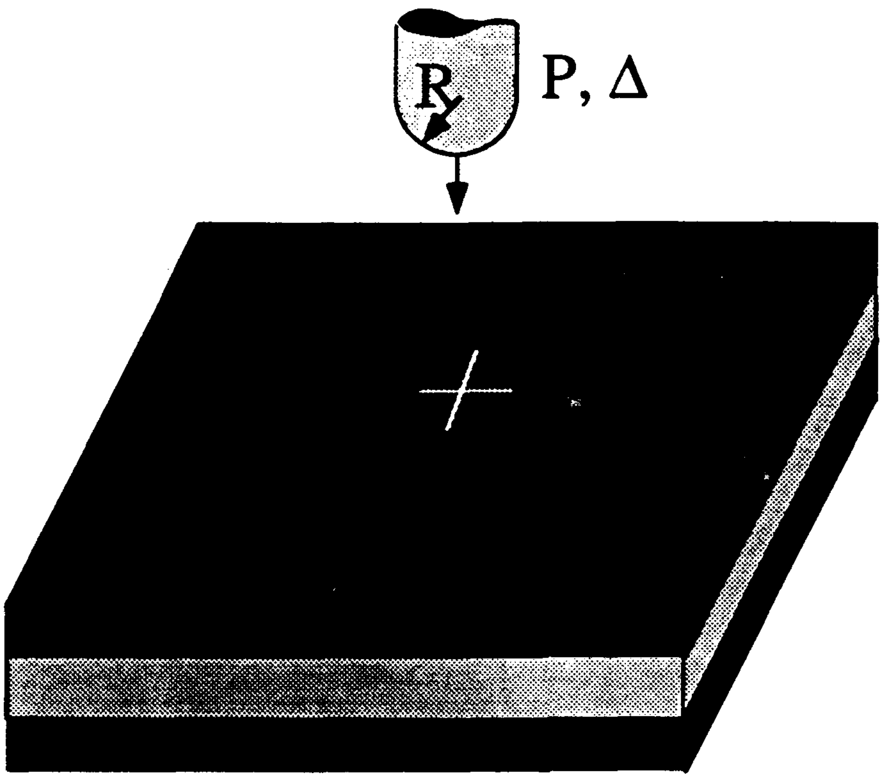

Figure 1. Description of the problem.

2-1

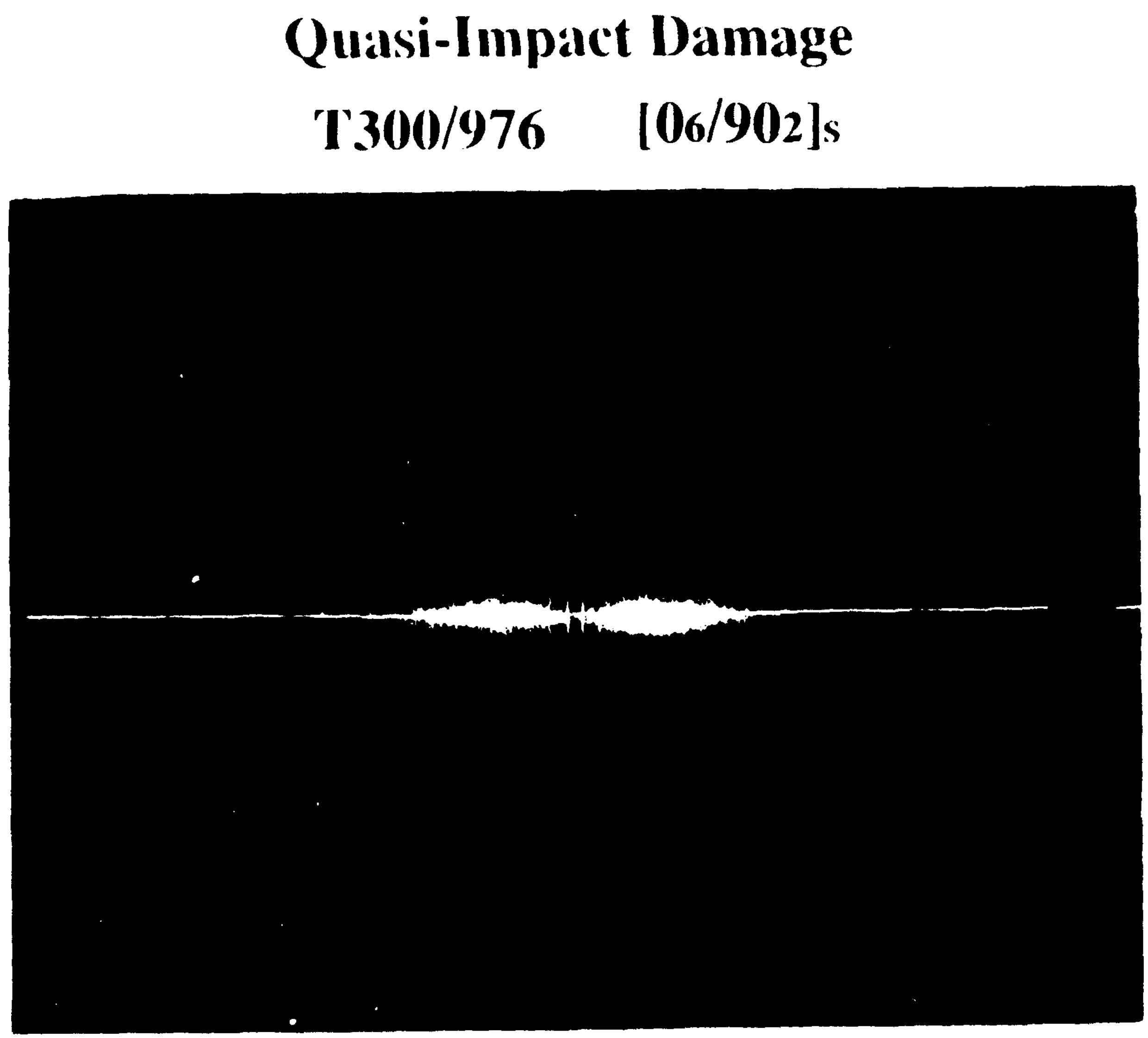

Figure 2. An X-Radiograph of a graphite/epoxy $[0_6/90_2]_s$ composite resulting from a quasi static transverse loading. Data taken from Ref. 5.

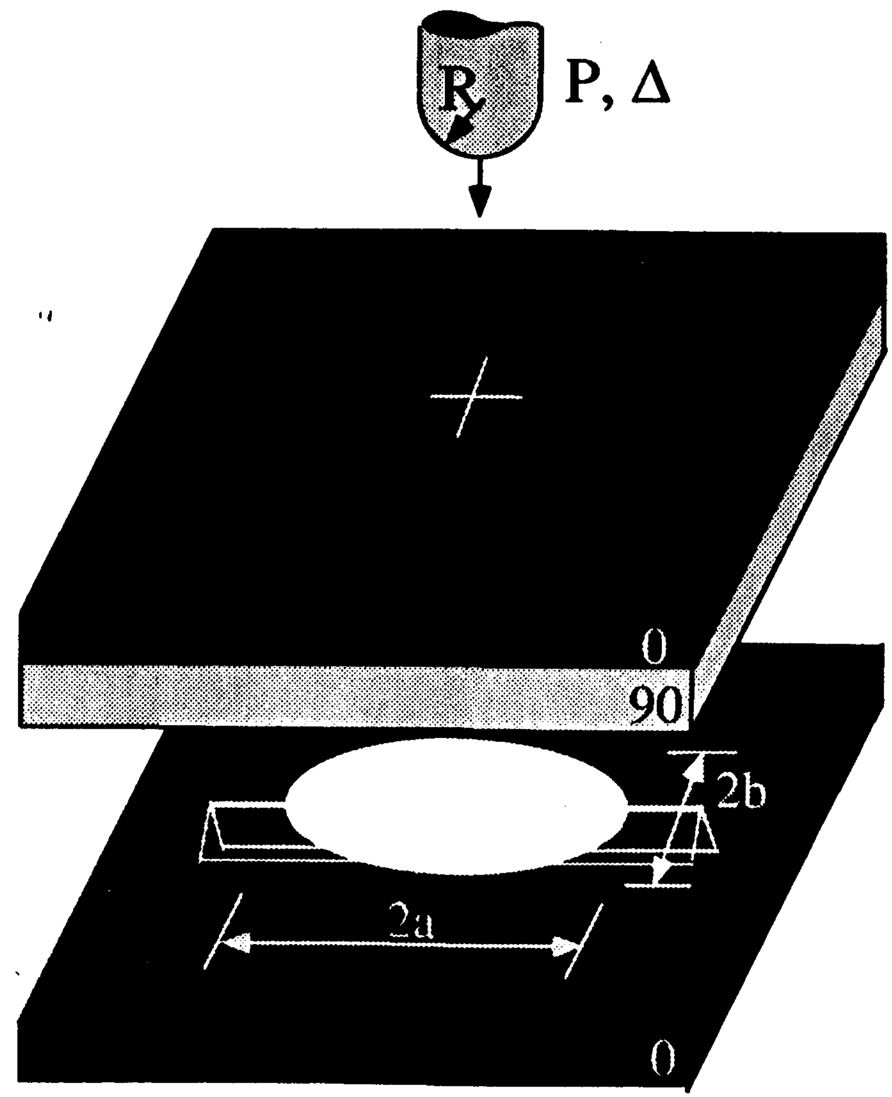

Figure 3. A delaminated cross-ply composite with or without a surface matrix and subjected to transverse loading from a spherical indenter.

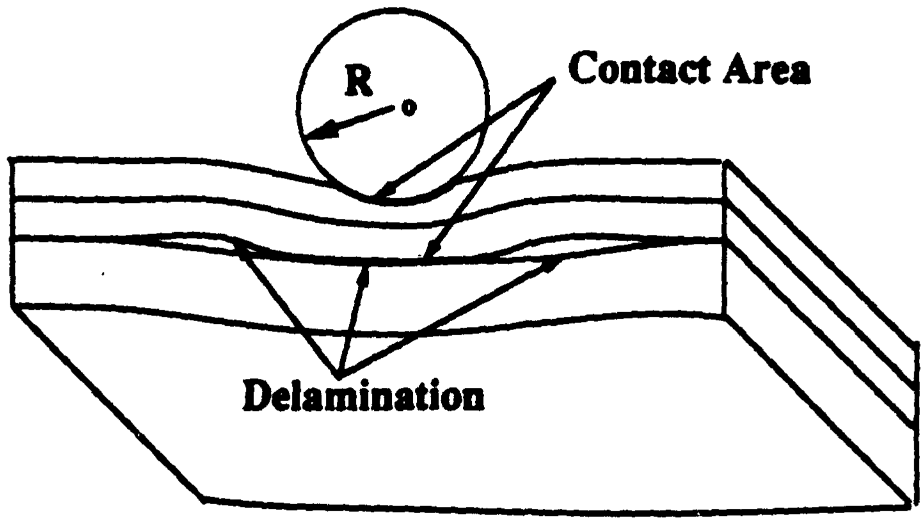

Figure 4. Description of the contact problem.

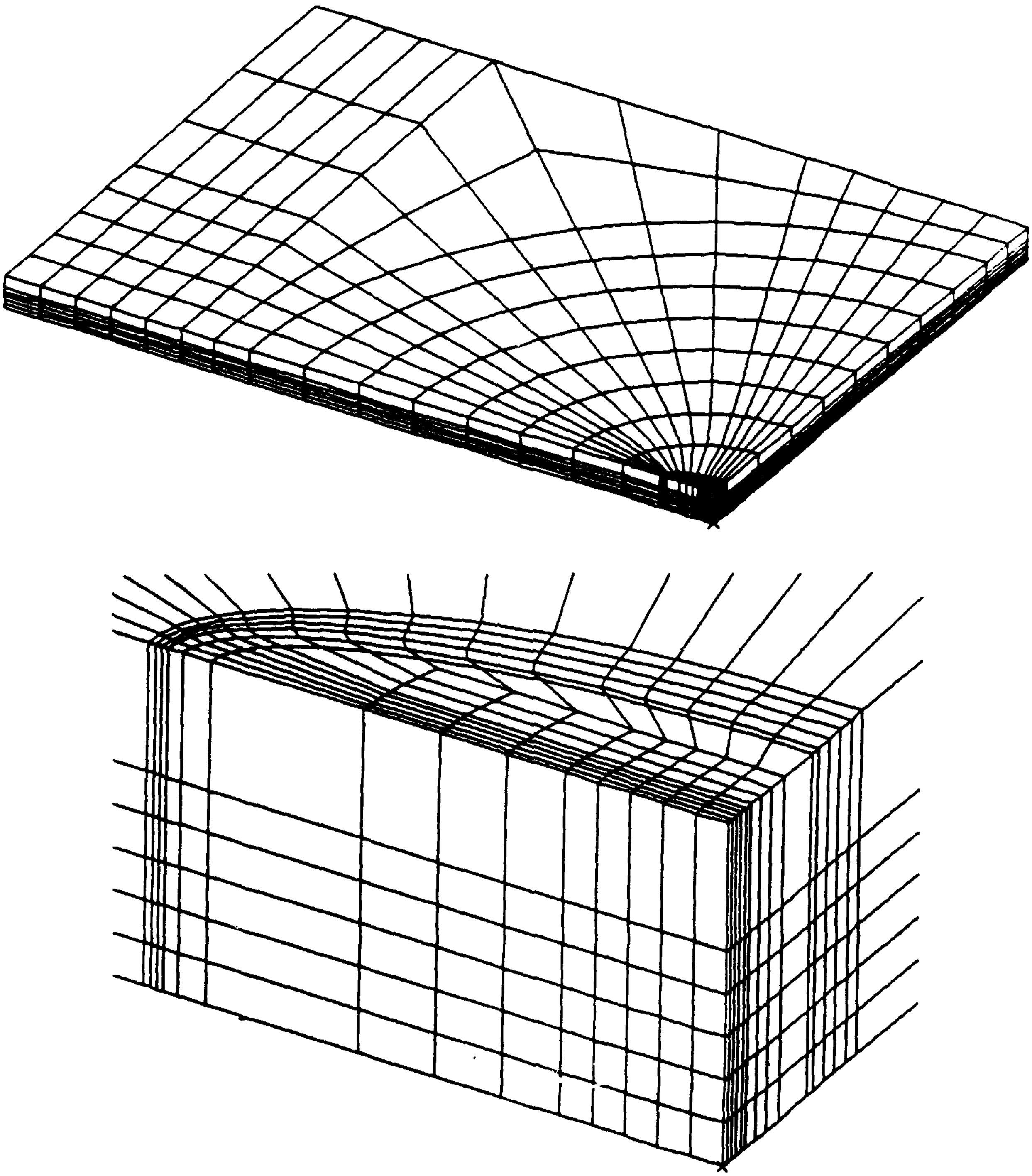

Figure 5. Description of a typical finite element mesh.

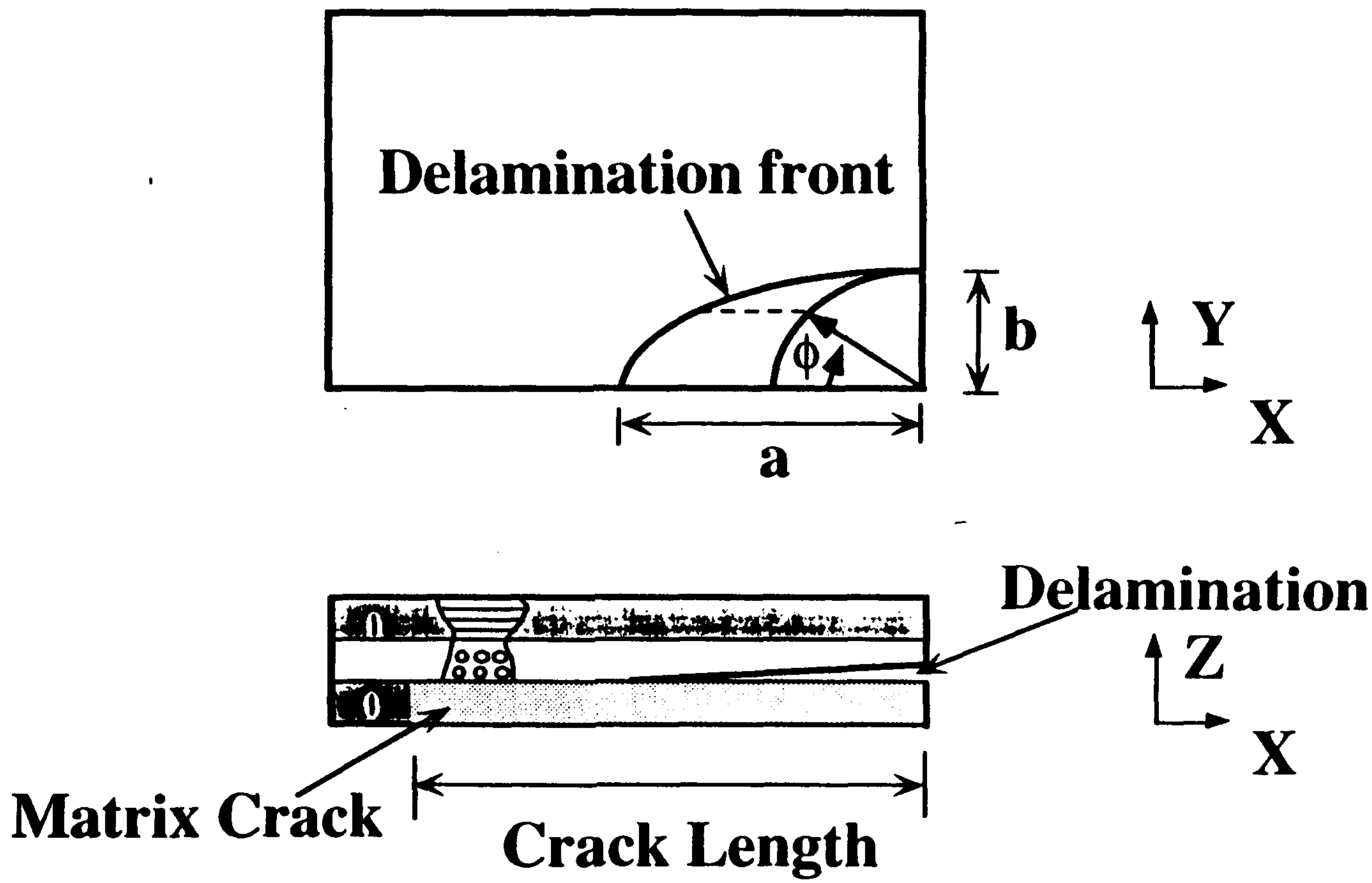

Figure 6. The coordinate system used for the delamination with a circular or an elliptical shape.

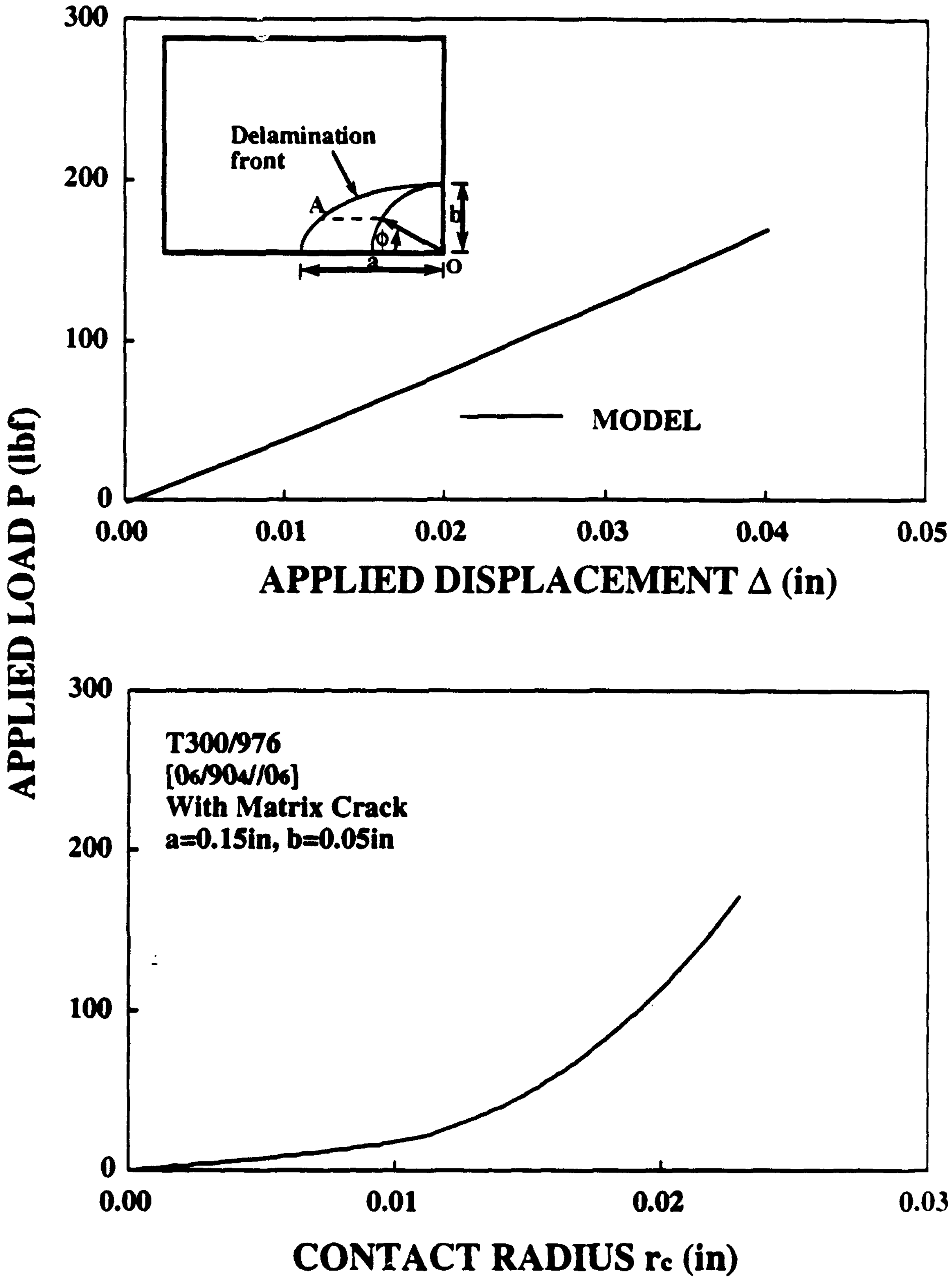

Figure 7. The responses of a $[0_6/90_2]_s$ composite containing a pre-introduced delamination and surface matrix crack and subjected to transverse loading resulting from a spherical indenter.

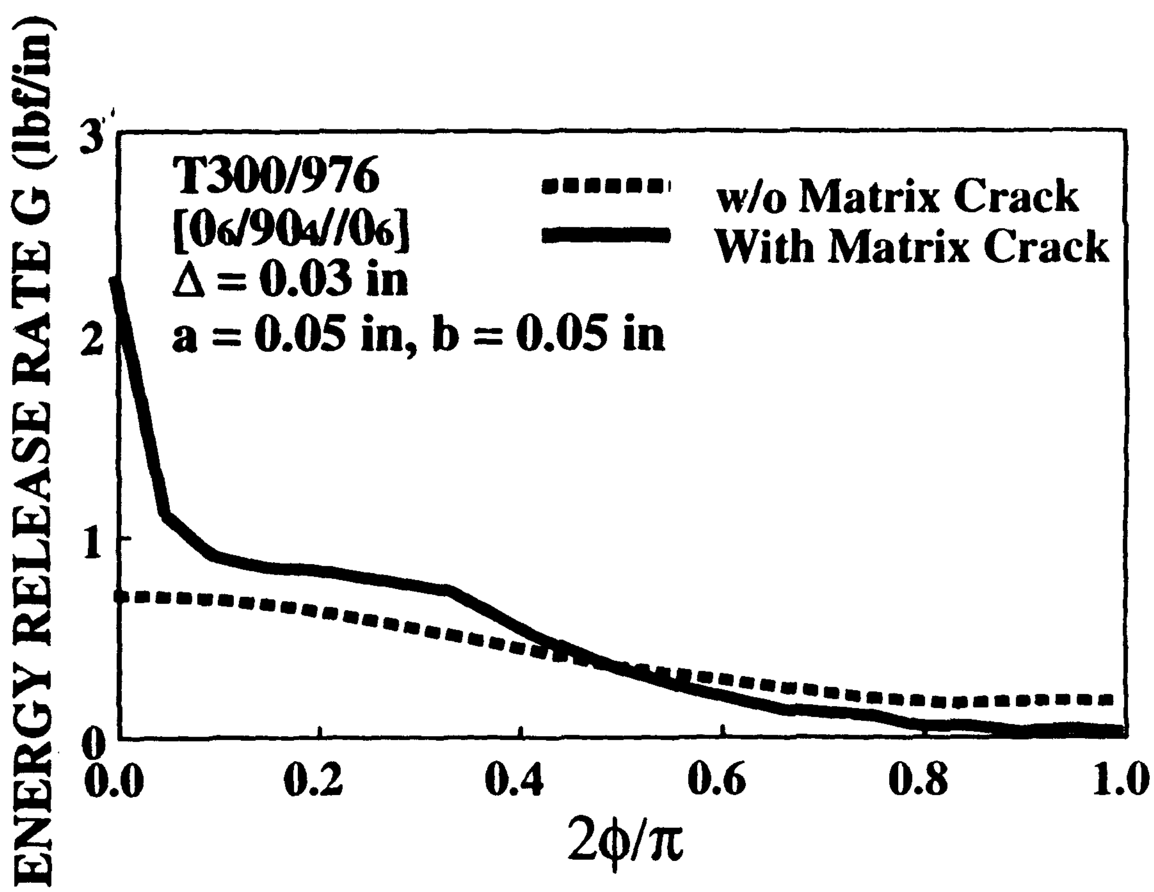

Figure 8. Comparison of the calculated total strain energy release rate along a delamination front in a $[0_6/90_2]_s$ composite with and without a surface matrix crack.

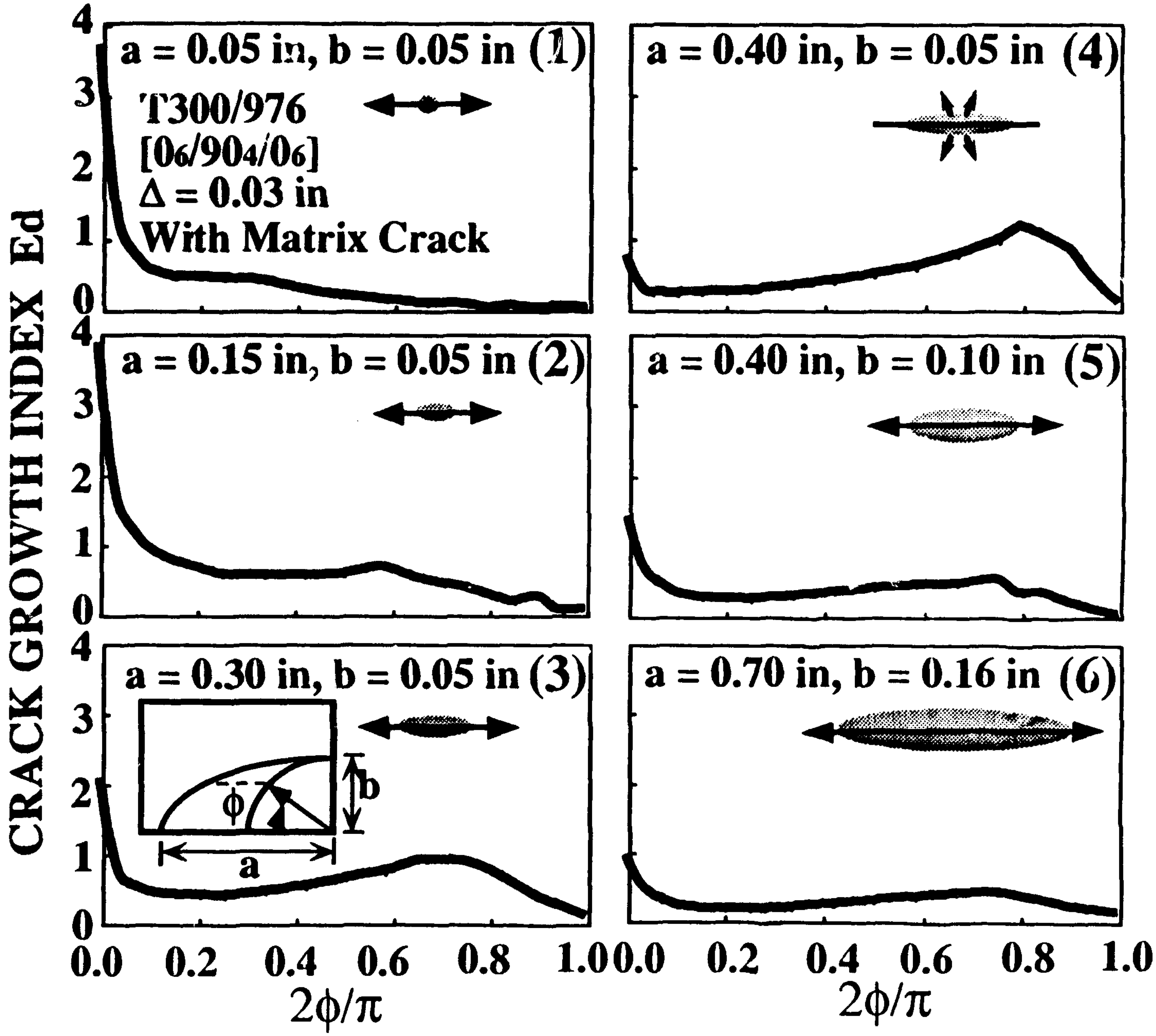

Figure 9. The predicted delamination growth sequence in a $[0_6/90_2]_s$ composite containing a surface matrix crack.

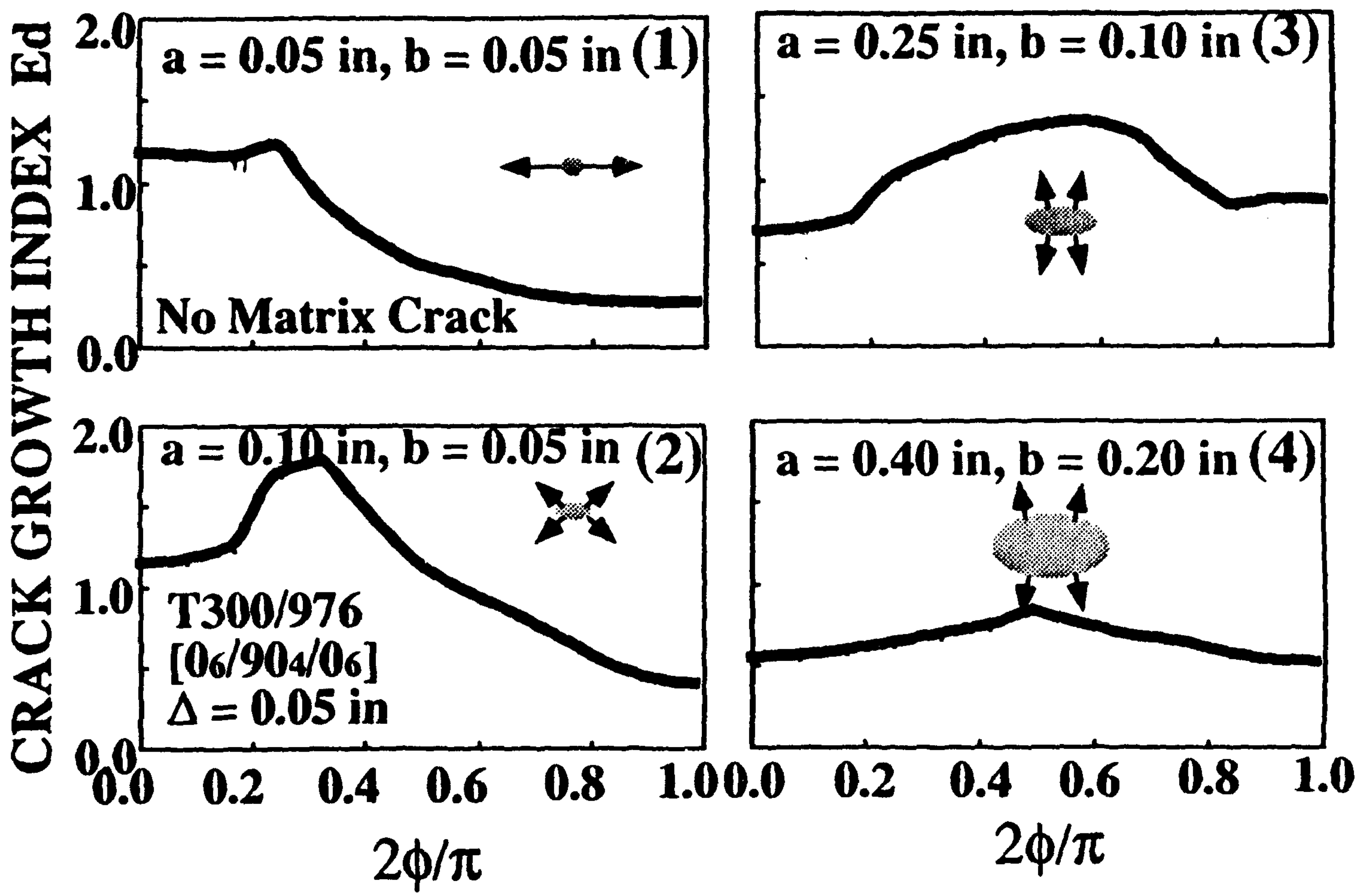

Figure 10. The predicted delamination growth sequence in a $[0_6/90_2]_s$ composite without the surface matrix crack.

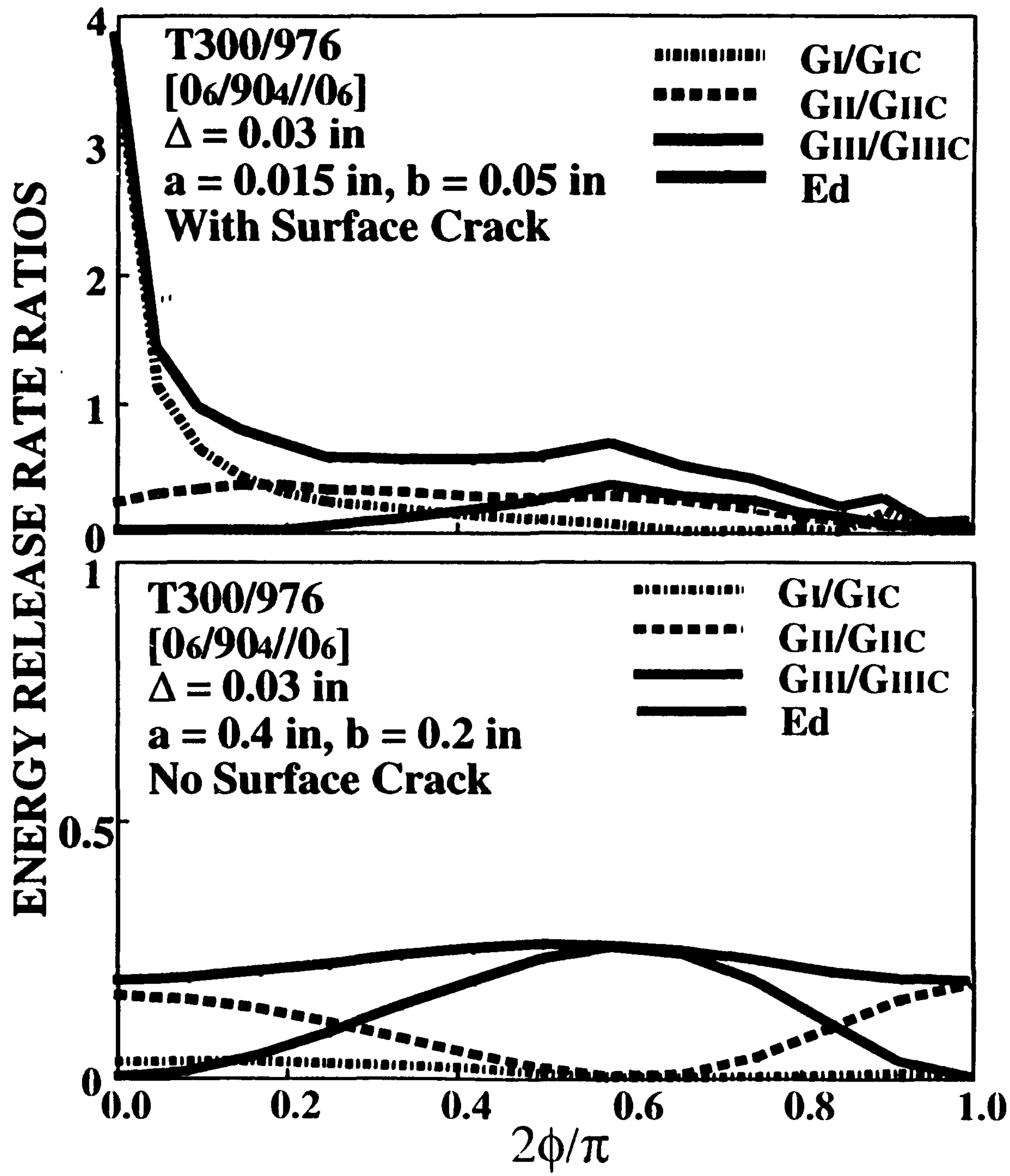

Figure 11. Comparison of the calculated strain energy release rates Modes I, II and III along a delamination front in a $[0_6/90_2]_s$ composite with and without a surface matrix crack.